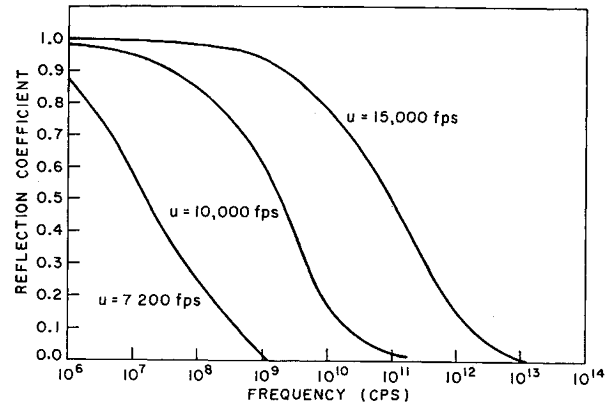

Fig. 3 Reflection coefficient vs electromagnetic wave frequency. Probe velocity = 7200, 10,000, and 15,000 fps; altitude = 200,000 ft

The reflection coefficient calculated for a semi-infinite shock layer, with a plane polarized normally incident em wave, indicates strong reflection for all radio frequencies (Fig. 3). The applicability of this model was examined by calculating the penetration depth to determine if there is sufficient shock layer thickness to establish reflection conditions. At radar frequencies, the penetration depths are the order of millimeters or less, whereas the shock layer thicknesses considered are 3 to 7 cm, indicating that the semi-infinite shock layer model is applicable. For 3-cm radar waves, penetration depths vary from 0.5 mm to 1 cm. At low frequencies the penetration depths are greater than the shock layer thickness considered, and so reflection losses are not predicted accurately by the forementioned model. It appears that low and high frequency signals could be used to transmit information from the probes, although intermediate frequencies (10^8 to 10^{11} cps) would not be available.

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A Vaneless Turbopump

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REFERENCE 1 deals with a method of direct energy exchange between flows, consisting of an essentially nondissipative interaction followed by complete mixing at constant pressure. In applications of the method to thrust or lift generation, the final mixing process has no effect on performance if it takes place at ambient pressure. In other situations, however, the effect of mixing on performance is neither negligible nor always beneficial, and there are applications in which it is required that mixing be avoided as much as possible and that the two flows be discharged separately, i.e., before they have mixed appreciably, from the interaction region. Two different ways of effecting this separation are suggested[2] by the observation that the two flows are steady in the relative frame and have different orientations, before they mix, in the stationary frame.

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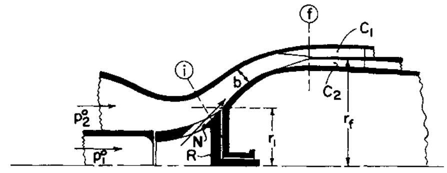

Fig. 1 A vaneless turbopump; *R* is free-spinning rotor, *N* a primary discharge nozzle, C_1 and C_2 stationary cascades of separator

This note is a continuation of Ref. 1 and deals specifically with a pumping application in which the two flows leave the interaction region through separate ports before transport processes across their interfaces have made significant progress. Figure 1 illustrates a situation of this type, in which the separation is effected through two stationary cascades C_1 and C_2, one of which is aligned to the primary flow and the other to the secondary flow. The notation will be consistent with that previously employed, and compressibility, viscosity, and heat conduction again will be neglected.

The two flows are extracted from the interaction space at station f, where $p_f = p_e$, $c_{1f}^2 = (1 + \lambda)u_1'^2$, $c_{2f}^2 = (P + \lambda)u_1'^2$, and $V_f^2 = \lambda u_1'^2$. Here, as in Ref. 1,

$$\lambda = \left(1 + \frac{p_e - p_i}{\frac{1}{2}\rho_1 u_1'^2}\right)\left(\frac{r_f}{r_i}\right)^2 \tan\beta_i$$

$$P = \frac{\rho_1}{\rho_2}\,\frac{1 - (p_e/p_0^0)}{(p_1^0/p_0^0) - (p_e/p_0^0)}$$

From the law of cosines

$$u_{2f}^2 = c_{2f}^2 + V_f^2 - 2c_{2f}V_f \sin\beta_f$$
$$= u_1'^2\left[P + 2\lambda\,\frac{(1 + \lambda)^{1/2} - (P + \lambda)^{1/2}}{(1 + \lambda)^{1/2} + \mu(P + \lambda)^{1/2}}\right]$$

Since $(p_{2f}^0 - p_e)/(p_1^0 - p_e) = \rho_2 u_{2f}^2/\rho_1 u_1'^2$, it follows that the compression ratio is

$$\frac{p_{2f}^0}{p_0^0} = 1 + 2\lambda\,\frac{\rho_2}{\rho_1}\,\frac{(1 + \lambda)^{1/2} - (P + \lambda)^{1/2}}{(1 + \lambda)^{1/2} + \mu(P + \lambda)^{1/2}}\left(\frac{p_1^0 - p_e}{p_0^0}\right)$$

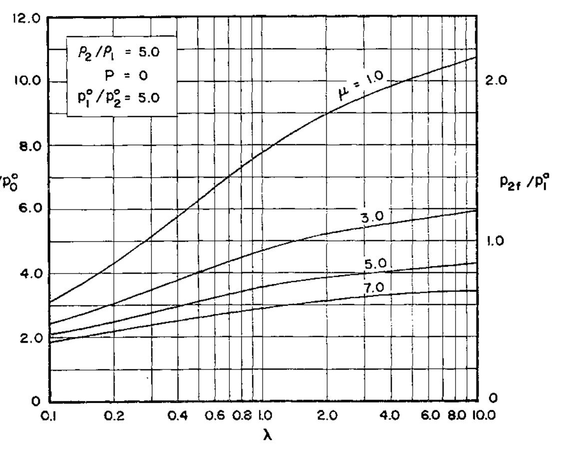

Fig. 2 Ideal compression ratio (with total separation)

This relation is plotted in Fig. 2. It is noteworthy that with moderate mass flow ratios the total pressure of the outflowing secondary flow can be considerably higher than that of the inflowing primary. This points to the possibility of promising looped arrangements (turbocompressors, gas generators, turbojets) in which some or all of the extracted secondary flow is energized through heat addition to form the primary source.

These results are in general agreement with those obtained by a different procedure in Ref. 2, for fully looped systems, with consideration of compressibility effects.

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Preliminary Orbit Determination for a Moon Satellite from Range-Rate Data

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Derivation

SEVERAL of the manned lunar missions planned for this decade require the establishment of selenocentric orbits. The data for determining these orbits may originate from 1) earth-based observations, 2) a base on the moon, 3) another vehicle on a definitive selenocentric orbit, or 4) the orbiting vehicle itself. Because of the large separation distances involved, the first alternative is, at present, impractical. Likewise, since neither a moon base nor a selenocentric satellite on a definitive orbit from which observations can be made will be established for the early missions, alternatives 2 and 3 are not considered. For these reasons, a method based upon the fourth alternative is presented here.

Preliminary determination of the position and velocity of a selenocentric orbiting vehicle is obtained from two measurements of range-rate (vertical to the lunar surface), vehicular sublatitude, sublongitude, and the time separation between these measurements. The determination of sublatitude and sublongitude points can be accomplished by recognizing lunar landmarks and associating them with their proper selenocentric coordinates and/or by use of stellar observations. The former method is, of course, greatly dependent upon progress in knowledge of the lunar cartography. The computations can be performed by either an on-board or earth-based computer. A mathematical model using two-body analysis throughout is described in this paper. Like the modified Gaussian procedure of orbit determination from two positions and time of flight developed by Herrick and Liu,[1–3] this method requires iteration upon assumed values of the parameter p (semilatus rectum). However, although in the former method a unique true anomaly is obtained for each assumed p, in this method the value of the true anomaly is independent of the parameter, a fact that results in a less complex iteration procedure.

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The following inertial coordinate system is adopted. The origin of coordinates is taken as the geometric center of the moon with the x and y axes lying in the lunar equator plane and the z axis directed toward the lunar north pole. The x axis is coincident with the intersection of the lunar prime meridian with the lunar equator at the time of the first observation, taken as positive toward the earth. The sense of the y axis is such that x, y, z forms a right-handed set (see Fig. 1).

The unit vectors $\mathbf{U}_1$ and $\mathbf{U}_2$ directed from the origin to the satellite at the observation times are given by

$$\mathbf{U}_1 = \cos\phi_1 \cos\lambda_1 \mathbf{i} + \cos\phi_1 \sin\lambda_1 \mathbf{j} + \sin\phi_1 \mathbf{k} \quad (1)$$

$$\mathbf{U}_2 = \cos\phi_2 \cos[\lambda_2 + \dot{\theta}_{☾}(t_2 - t_1)]\mathbf{i} + \cos\phi_2 \sin[\lambda_2 + \dot{\theta}_{☾}(t_2 - t_1)]\mathbf{j} + \sin\phi_2 \mathbf{k} \quad (2)$$

where $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$ are unit vectors in the x, y, z directions, respectively, $\dot{\theta}_{☾}$ is the rotation rate of the moon, ϕ and λ represent the lunar sublatitude and sublongitude, respectively, t is the observation time, and the subscripts 1 and 2 correspond to the first and second observations. For this analysis the longitude is measured as positive in the direction of the moon's rotation with 0° longitude located along the prime meridian.

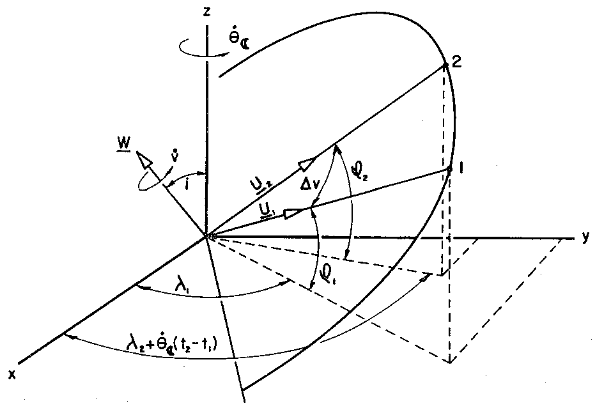

Fig. 1 Inertial coordinate system

The increment in true anomaly Δv between the forementioned vectors is given by the dot product

$$\mathbf{U}_1 \cdot \mathbf{U}_2 = \cos(v_2 - v_1) = \cos\Delta v = \cos\phi_1 \cos\phi_2 \cos\lambda_1 \cos[\lambda_2 + \dot{\theta}_{☾}(t_2 - t_1)] + \cos\phi_1 \cos\phi_2 \sin\lambda_1 \sin[\lambda_2 + \dot{\theta}_{☾}(t_2 - t_1)] + \sin\phi_1 \sin\phi_2 \quad (3)$$

For this analysis, the assumption is made that $0 < \Delta v < \pi$. In view of the present lack of knowledge of landmarks on the back side and limbs of the moon, this assumption is felt to be valid. Thus Δv may be assigned to the first or second quadrant and uniquely determined from Eq. (3).

The radial velocities $\dot{r}$ at the observation times satisfy the following relations:

$$\dot{r}_1 = (\mu/p)^{1/2}\, e \sin v_1 \quad (4a)$$

$$\dot{r}_2 = (\mu/p)^{1/2}\, e \sin(v_1 + \Delta v) \quad (4b)$$

where e is the orbital eccentricity, p is the parameter (semilatus rectum), and μ is the product of the universal gravitational constant and the mass of the moon or the product of the gravitational acceleration at the moon's surface and the square of the (assumed constant) lunar radius. Since the triaxiality of the moon and local anomalies are neglected in this analysis, the measured range-rate may be set equal to $\dot{r}$. From Eqs. (4a) and (4b) one can obtain, after some manip-